

Chapter 1

1.1 Seven degrees of freedom. An example of the structural model is shown in figure E.1.

1.2 The singular configuration is shown in figure E.2.

1.3 Five degrees of freedom.

Chapter 2

2.1 If $\cos\theta \neq 0$, then $\phi = \text{atan2}(\pm R_{21}, \pm R_{11})$, $\theta = \text{atan2}(-R_{31}, \pm\sqrt{R_{32}^2 + R_{33}^2})$, and $\psi = \text{atan2}(\pm R_{32}, \pm R_{33})$. If $\cos\theta = 0$, then ϕ is arbitrary, $\theta = -R_{31} \times 90^\circ$, and $\psi = \text{atan2}(R_{12}, R_{22}) + R_{31}\phi$.

2.2 From equations 2.23a and 2.23d, $\sin\psi = -\sin\phi R_{11} + \cos\phi R_{21}$; from equations 2.23b and 2.23e, $\cos\psi = -\sin\phi R_{12} + \cos\phi R_{22}$. Thus, $\psi = \text{atan2}(-\sin\phi R_{11} + \cos\phi R_{21}, -\sin\phi R_{12} + \cos\phi R_{22})$.

2.5 $(\phi, 180^\circ, 120^\circ + \phi)$, where ϕ is arbitrary.

2.7 Transform T_p gives the image of an original point when a convex lens with focal distance f is placed on the X - Z plane.

2.11 $r_x = l_1 C_1 + l_2 C_{12}, r_y = l_1 S_1 + l_2 S_{12}, r_z = d_3, \alpha = \theta_1 + \theta_2 + \theta_4.$

2.16 $\tau = [0, -100(l_c + l_e), -100l_e, 0, 0, 0]^T.$

2.17 ${}^A x_B^T {}^A z_B = {}^A x_B^T ({}^A x_B \times {}^A y_B) = {}^A y_B^T ({}^A x_B \times {}^A x_B) = 0,$
 $1 = {}^A z_B^T {}^A z_B = {}^A x_B^T [{}^A y_B \times ({}^A x_B \times {}^A y_B)]$
 $= {}^A x_B^T [({}^A y_B^T {}^A y_B) {}^A x_B - ({}^A y_B^T {}^A x_B) {}^A y_B] = 1 - ({}^A y_B^T {}^A x_B)^2.$

Chapter 3

3.1 $I = \int_V [r^T r \mathbf{I}_3 - r r^T] \rho dv = \int_V [{}^B r^T {}^B r \mathbf{I}_3 - {}^V R_B {}^B r {}^B r^T ({}^V R_B)^T] \rho dv$
 $= {}^V R_B {}^B I ({}^V R_B)^T.$

3.2 $\tau_1 = (m_1 l_{g1}^2 + \tilde{I}_1 + m_2 d_2^2 + \tilde{I}_2) \ddot{\theta}_1 + 2m_2 d_2 \dot{d}_2 \dot{\theta}_1$
 $+ (m_1 l_{g1} + m_2 d_2) C_1 g,$
 $\tau_2 = m_2 \ddot{d}_2 - m_2 d_2 \dot{\theta}_1^2 + m_2 S_1 g.$



Figure E.1
Seven-joint manipulator.

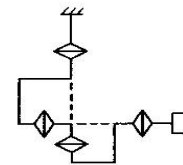


Figure E.2
Singular configuration of four-joint wrist.

3.7 $\tau_1 = (m_a l_{ga}^2 + \tilde{I}_a + m_c l_{gc}^2 + \tilde{I}_c + m_d l_a^2) \ddot{\theta}_1$
 $- (l_{gc} l_b m_c - l_a l_{gd} m_d) \cos(\theta_1 - \theta_2) \ddot{\theta}_2$
 $- (l_{gc} l_b m_c - l_a l_{gd} m_d) \sin(\theta_1 - \theta_2) \dot{\theta}_2^2$
 $+ (l_{ga} m_a + l_{gc} m_c + l_a m_d) g C_1,$
 $\tau_2 = (m_b l_{gb}^2 + \tilde{I}_b + m_c l_b^2 + m_d l_{gd}^2 + \tilde{I}_d) \ddot{\theta}_2$
 $+ (m_c l_{gc} l_b - m_d l_a l_{gd}) \cos(\theta_1 - \theta_2) \ddot{\theta}_1$
 $+ (m_c l_{gc} l_b - m_d l_a l_{gd}) \sin(\theta_1 - \theta_2) \dot{\theta}_1^2 - (m_b l_{gb} + m_c l_b - m_d l_{gd}) g C_2.$

3.8 $\phi_d^* = [\phi_{d1}, \dots, \phi_{d5}]^T$, where $\phi_{d1} = m_a l_{ga}^2 + \tilde{I}_a + m_c l_{gc}^2 + \tilde{I}_c + m_d l_a^2,$
 $\phi_{d2} = m_b l_{gb}^2 + \tilde{I}_b + m_c l_b^2 + m_d l_{gd}^2 + \tilde{I}_d,$ $\phi_{d3} = l_{gc} l_b m_c - l_a l_{gd} m_d,$ $\phi_{d4} =$
 $m_a l_{ga} + m_c l_{gc} + m_d l_a,$ $\phi_{d5} = m_b l_{gb} + m_c l_b - m_d l_{gd}.$

Chapter 4

4.3 The singular values are

$$\sigma_i = [(a_1 \pm \sqrt{a_1^2 + 4a_0})/2]^{1/2},$$

where

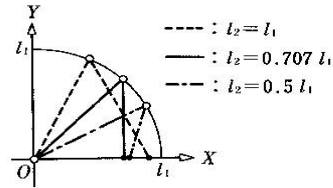


Figure E.3
Solution to exercise 4.3.

$$a_1 = l_1^2 + 2l_1 l_2 C_2 + 2l_2^2$$

and

$$a_0 = l_1^2 l_2^2 S_2^2.$$

The arm posture maximizing w_2 is given by θ_2 satisfying

$$C_2 = -2l_1 l_2 / (l_1^2 + 2l_2^2).$$

The optimal arm posture is shown in figure E.3 for various ratios of l_1 and l_2 .

4.4 $w = |\det J| = l_1 l_2 |\sin(\theta_2 - \theta_1)|$. The optimal arm posture is given by $\theta_2 - \theta_1 = \pm 90^\circ$.

$$\sigma_i = \sqrt{\lambda_i},$$

where

$$\lambda_i = \{(l_1^2 + l_2^2) \pm [(l_1^2 + l_2^2)^2 - 4l_1^2 l_2^2 \sin^2(\theta_2 - \theta_1)]^{1/2}\} / 2$$

($i = 1$ corresponds to $+$, and $i = 2$ corresponds to $-$).

Chapter 5

5.4 $k_f = [-0.358, 0.358, 0.862]^T$, $\alpha_f = 81.6^\circ$.

5.7 Let $\tilde{t} = (\sigma \omega_c^2)^{1/3} t$ and let $C = \sqrt{\sigma^2 - 2\sigma\zeta\omega_c + \omega_c^2}$. Then $y(\tilde{t}) = 1 - A \exp(-\sigma\tilde{t}) + AB \exp(-\zeta\omega_c\tilde{t}) \sin(\sqrt{1 - \zeta^2}\omega_c\tilde{t} + \theta)$,

where $A = \omega_c^2 / C^2$, $B = \sigma C / (\omega_c^2 \sqrt{1 - \zeta^2})$, and $\sin\theta = (2\zeta\omega_c - \sigma) \sqrt{1 - \zeta^2} / C$.

$$\mathbf{5.8} \quad \mathbf{u} = -[x_1^2 - 2x_1x_2 - 4x_2^2 + 3x_1 + x_2 + \lambda_1 \tilde{u}_1 - \lambda_2 x_2 \tilde{u}_2, 2x_1 + 4x_2 + \lambda_2 \tilde{u}_2]^T.$$

Chapter 6

6.1 $m_d = 1.25$, $k_d = 625$, $d_d = 57.5$.

6.5 Let $\mathbf{r} = [x, y, z, \phi, \theta, \psi]^T$, where x , y , and z are the X , Y , and Z coordinates and ϕ , θ , and ψ are Euler angles of the end effector with respect to the reference frame. Then the constraint surfaces are given by $l^2 - x^2 - y^2 = 0$, $z - z_0 = 0$, $\theta - \theta_0 = 0$, and $\psi - \psi_0 = 0$, where z_0 , θ_0 , and ψ_0 are constants.

6.7 Letting $u_x = \partial u / \partial x$, $u_{xy} = \partial^2 u / (\partial x \partial y)$, etc., we have

$$E_F = \begin{bmatrix} -u_x & 1 & -u_z & 0 & 0 & 0 \\ -u_{xx} & 0 & -u_{xz} & 1 + u_z^2 & 0 & 0 \\ \delta_1 & 0 & \delta_2 & 0 & \delta_3 & 0 \end{bmatrix},$$

where

$$\delta_1 = (1 + u_x^2)u_{xz} - u_x u_z u_{xx},$$

$$\delta_2 = (1 + u_x^2)u_{zz} - u_x u_z u_{xz},$$

and

$$\delta_3 = (1 + u_x^2 + u_z^2)(1 + u_x^2)^{1/2}.$$

Matrix E_p is, for example, given by

$$E_p = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Chapter 7

7.1 Show that $A = CB$ and $A^+ = B(CB)^+$ satisfy equations A2.1–A2.4 in appendix 2. Use the fact that $(I - J_1^+ J_1)$ is symmetric and idempotent.

7.2 Substitute equations 7.5 and 7.6 into equation 7.17, and let the derivative of p_1 with respect to k_1 be equal to $\mathbf{0}$.

7.3 Substitute equations 7.5 and 7.6 into equation 7.18, and let derivative of p_2 with respect to k_1 be equal to 0. Use

$$(\mathbf{I} - \mathbf{J}_1 + \mathbf{J}_1)^T (\mathbf{I} - \mathbf{J}_1 + \mathbf{J}_1) = (\mathbf{I} - \mathbf{J}_1 + \mathbf{J}_1).$$

7.7 A solution method is to specify the second subtask by the performance criterion

$$V(\mathbf{q}) = \sum_{i=1}^3 \{1/(\theta_{i,\max} - \theta_i)^2 + 1/(\theta_i - \theta_{i,\min})^2\},$$

which is to be kept small, and to obtain $\dot{\mathbf{q}}_d$ by the approach in subsection 7.2.4.